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New reduction strategy in the biobjective knapsack problem

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Abstract

In this paper, the admissible region of a biobjective knapsack problem is our main interest. Although the reduction of feasible region has been studied by some authors, yet more investigation has to be done in order to deeply explore the domain before solving the problem. We propose, however, a new technique based on extreme supported efficient solutions combined with the dominance relationship between items' efficiency. An illustration of the algorithm by a didactic example is given and some experiments are presented, showing the efficiency of the procedure compared to the previous techniques found in the literature.

Keywords: efficient solution; knapsack problem; regular variable; multiple objective

1. Introduction

The biobjective knapsack problem (BOKP), is an NP-hard combinatorial optimization problem (cf. Martello and Toth, 1990; Kellerer et al., 2004). An instance of BOKP is characterized by a set of n items and a knapsack capacity ω, where each item i, $i = 1, \ldots, n$, has a weight w_i and a distinguished profit c_i^k according to the objective functions $k = 1, 2$. Unlike the classical knapsack problem (KP) that optimizes one objective function, BOKP disposes of two objective functions to be optimized simultaneously. In this case, the goal of BOKP is to determine a set of objects to put in a bag, each object having a weight and a profit, the objects should be selected to maximize two functions without exceeding the capacity ω. Numerous algorithms have been designed to solve such problem, either based on implicit enumeration methods, such as dynamic programming, branch and bound, or apply heuristic procedures, especially metaheuristics, to approximate the set of efficient solutions that its size can grow exponentially with the number of items in the knapsack.

This latter is a motivation for exploring the information in the admissible region and finding a simple technique that reduces as much as possible the admissible domain.

Mathematically, BOKP can be stated as follows:

$$\text{(BOKP)} \begin{cases} \max(Z^k) = \sum_{i=1}^{n} c_i^k x_i, & k = 1, 2 \\ \sum_{i=1}^{n} w_i x_i \leq \omega \\ x_i \in \{0, 1\}, & i = 1, \ldots, n, \end{cases} \tag{1}$$

where x_i is a decision variable, equals to 1 if the item i is in the knapsack and 0 otherwise. For the rest of the paper and without loss of generality, we assume that

1. All input data ω, c_i, and w_i, for $i = 1, \ldots, n$, are nonnegative integers.
2. $\sum_{i=1}^{n} w_i > \omega$, in order to avoid trivial solutions.

2. Related works

BOKP belongs to the well-known knapsack family (cf. Kellerer et al., 2004) that represents a natural combinatorial optimization problems. BOKP is a more complex variant of the classical NP-hard single object KP and has a wide range of applications such as capital budgeting (cf. Dyer et al., 1992) and media selection (cf. Rosenblatt and Sinuany-Stern, 1989).

As far as we know, many papers addressing the BOKP are available, as well as the methods devoted to the classical single-objective knapsack. Among existing papers tackling the BOKP, a two-phase resolution search was proposed in Visée et al. (1998). The first phase provides the set of supported efficient solutions by solving a weighted sum objective functions and the second phase, with its several versions, is used to find the nonsupported efficient solutions.

Przybylski (2006) generalized the previous method with three objectives using the ranking method for searching for the efficient solutions in the second phase. In a such method, the second phase is very important since the nonsupported efficient solutions are to be found. The reason why many researchers have spent considerable time to find methods to solve in reasonable CPU-time. Dichotomy, ranking, implicit enumeration, and other techniques are often used.

The presence of more than one objective makes it more difficult. Moreover, large-scale optimization problems are time consuming, therefore reducing their size is necessary.

Dantzig (1957) showed that an optimal solution for the continuous {0, 1}-KP could be obtained as follows: the items are sorted according to their nonincreasing profit to weight ratios, and these items are included one by one without exceeding the knapsack capacity. In the end, only one item cannot be completely included in the KP; it is called the break item (critical item).

Balas and Zemel (1980) were the first to observe that for random instances, the optimal solution for the {0, 1}-KP is very similar to the continuous optimal solution. This similarity led to the introduction of the core concept.

Puchinger et al. (2006) presented the new core concept for the multidimensional KP (MKP), extending the core concept for the classical one-dimensional {0, 1}-KP.

The nonincreasing order values of the object efficiency (profit-to-weight) are not verified in the biobjective case. Gomes da Silva et al. (2008) studied the notion of core problems for the MOKP, but they used a family of weighted sum objective functions, where each function of this family is considered separately.

Mavrotas et al. (2009) also defined the core concept for the multiobjective MKP and developed a method based on the core concept for the biobjective case. In the latter works, the main computing time effort was devoted to search the exact core that depends on the efficient solution considered.

Jorge et al. (2008) presented new properties aiming to reduce the size of the biobjective BOKP. Based on a lower bound (LB) and upper bound (UB) on the cardinality of a feasible solution for the KP introduced by Glover (1965) and on dominance relations in the data space of the MOKP, they reduced the size of the biobjective instances of the BOKP by fixing a priori about 10% of the variables, on random instances.

The remainder of the paper is organized as follows. Some basic definitions are presented in Section 3 followed by Section 4, where our developed algorithm is detailed and illustrated by a didactic example. In Section 5, some numerical results are given for comparing the results in Jorge et al. (2008) and the existing other results. Finally, in Section 6, a conclusion and some remarks are presented.

3. Theoretical presentation of the problem

In a multiobjective framework, the KP can be formulated as:

$$(MOKP)\begin{cases}\max(Z^k) = \sum_{i=1}^{n} c_i^k x_i \quad k = \overline{1, p} \\ \sum_{i=1}^{n} w_i x_i \leq \omega \\ \qquad x_i \in \{0, 1\} \quad i = \overline{1, n},\end{cases} \tag{2}$$

where

1. c_i^k, w_i, and w are positive integers and $x = (x_1, x_2, \ldots, x_n)$ in order to avoid trivial solutions, we suppose in addition that
2. $w_i \leq \omega, \quad i = \overline{1, n}$.
3. $\sum_{i=1}^{n} w_i > \omega$.

Definition 1. *A feasible solution x^* is efficient for* (2)*, if there does not exist any other feasible solution x such that $Z^k(x) \geq Z^k(x^*)$, $k = \overline{1, p}$ with at least one strict inequality.*

Definition 2. *The vector $Z(x^*) = (Z^1(x^*), Z^2(x^*), \ldots, Z^p(x^*))$ is said to be nondominated in the space of objective functions.*

The presence of integer decision variables imposes two incompatible sets of efficient solutions:

- The set $SE(MOKP)$ of supported efficient solutions that are optimal solutions of the parameterized single-objective problem.
- The set $NSE(MOKP) = E(MOKP) \backslash SE(MOKP)$ of nonsupported efficient solutions that cannot be found by optimization of the parameterized single-objective problem.

Definition 3. *An LB and UB on the cardinality of a feasible solution for the unidimensional KP was introduced by Glover (1965).*

These bounds are defined, respectively, as follows:

$$LB = \max \left\{ s : \sum_{i=1}^{s} w_i \leq \omega \right\} \quad \text{such that } w_i \geq w_{i+1}, \forall i \in \{1, \ldots, n-1\} \tag{3}$$

$$UB = \max \left\{ s : \sum_{i=1}^{s} w_i \leq \omega \right\} \quad \text{such that } w_i \leq w_{i+1}, \forall i \in \{1, \ldots, n-1\}. \tag{4}$$

Remark 1. LB and UB are independent from objective functions. Gandibleux and Fréville (2000) generalized their use in the case of multicriteria through the following proposal:

Proposition 1. *Let X_E be the complete set of efficient solutions of (2). If $x \in X_E$, then*

$$LB \leq \sum_{i=1}^{n} x_i \leq UB.$$

Definition 4. *Object's efficiency is defined by the ratio of the profit to the weight denoted by $e_i = \frac{c_i}{w_i}$.*

Remark 2. The notion of object's efficiency and efficient solution are completely different.

3.1. Properties of the regular variables (Jorge et al., 2008)

Definition 5. (Regular variables). *Let X_E be the complete set of efficient solutions of (2).*

$$C_0 = \left\{ i \in \{1, \ldots, n\} \mid x_i = 0, \ \forall x \in X_E \right\}$$
$$C_1 = \left\{ i \in \{1, \ldots, n\} \mid x_i = 1, \ \forall x \in X_E \right\}.$$

The regular variables are the components that contain the same value as that of all the efficient solutions.

Data dominance

The data space vectors of the KP :

$$V = \left\{ v^i \in \mathbb{N}^n \times \mathbb{Z} \,\middle|\, v^i = \left(c_i^1, \dots, c_i^p, -w_i\right), i \in \{1, \dots, n\} \right\}.$$

Definition 6. *Consider $v^i, v^j \in V$. A vector v^i dominate v^j ($v^i \succ v^j$), if and only if $(c_i^1, \dots, c_i^p, -w_i) \geq (c_j^1, \dots, c_j^p, -w_j)$ with at least one strict inequality.*

Definition 7. *Let $v^i \in V$. The set of all indices j of vectors v^j that are preferred to the vector v^i*

$$P(v^i) = \{ j \in \{1, \dots, n\} : v^j \succ v^i \}$$

is called the dominant (preferred) set.

Definition 8. *Let $v^i \in V$. The set of all indices j of vectors v^j that are dominated by the vector v^i*

$$D(v^i) = \{ j \in \{1, \dots, n\} : v^i \succ v^j \}$$

is called the dominated set.

The necessary conditions for regular variable are given in Jorge et al. (2008):

- if $|P(v^i)| \geq UB$ or $\sum_{j \in P(v^i)} w_j + w_i > w$ then $C_0^{'} = C_0^{'} \bigcup \{i\}$.
- if $n - |D(v^i)| \leq LB$ or $\sum_{j \notin D(v^i)} w_j < w$ then $C_1^{'} = C_1^{'} \bigcup \{i\}$.

$C_0^{'}$ and $C_1^{'}$ are the regular variables.

The proposed approach is mainly based on the notion of efficiency stated above (see Definition 4). Let C_0 and C_1 be the indices set corresponding to the regular variables. Consider the set with vectors E^i of components e_i^k, $i = 1, \dots, n$; $k = 1, \dots, p$. Thus $E = \{E^i \in \mathbb{R}^n : E^i = (e_i^1, e_i^2, \dots, e_i^p), i \in \{1, \dots, n\}\}$, where $e_i^k = \frac{c_i^k}{w_i}$ $k = 1, \dots, p$. The notion of dominance in the set E is defined in the same classical way by the following definition:

Definition 9. *Let $E^i, E^j \in E$. A vector E^i dominate E^j ($E^i \succ E^j$), if and only if $(e_i^1, \dots, e_i^p) \geq (e_j^1, \dots, e_j^p)$ with at least one strict inequality.*

Definition 10. *Let $E^i \in E$. The set of all indices j of vectors E^j that are preferred to the vector E^i*

$$P(E^i) = \{ j \in \{1, \dots, n\} : E^j \succ E^i \}$$

is called the preferred set .

Definition 11. *Let $E^i \in E$. The set of all indices j of vectors E^j that are dominated by the vector E^i*

$$D(E^i) = \{j \in \{1, \dots, n\} : E^i \succ E^j\}$$

is called the dominated set.

4. Description of the algorithm

We briefly describe the different steps of the procedure. The proposed algorithm is articulated on two procedures, the first one, determines p extreme supported efficient solutions, the second however, discards some regular variables from the admissible set.

We denote by $X^1, X^2, \dots, X^p$, the extreme supported efficient solutions of the problem $MOKP(2)$ such that $X^k = (x_1^k, x_2^k, \dots, x_n^k)$ $k = 1, 2, \dots, p$.

We also define the sets J_0 and J_1 by

$$J_0 = \left\{j \in \{1, \dots, n\} / x_j^1 = x_j^2 = \cdots = x_j^p = 0\right\} \tag{5}$$

$$J_1 = \left\{j \in \{1, \dots, n\} / x_j^1 = x_j^2 = \cdots = x_j^p = 1\right\} \tag{6}$$

Without loss of generality, we will consider the BOKP ($p = 2$).

- To find two extreme supported efficient solutions X^1 and X^2 of BOKP, we propose the following lexicographic method:

  - The lexicographic optimal solution X^1 corresponding to $lexmax\big(Z^1(x), Z^2(x)\big)$: initially, we solve the first objective $Z^1(x)$, with parameter set equals to $(1, 0)$, let X_{opt}^1 be an optimal solution. If it is unique then the optimal solution is efficient and the lexicographic optimal solution X^1 is then set to X_{opt}^1. Otherwise, if there exists many optimal solutions, then X_{opt}^1 can be a weakly efficient solution, in this case, to improve the second objective without degrading the first objective, we optimize the second single-objective function $Z^2(x)$ under the constraint $Z^1(x) = Z^1(X_{opt}^1)$. Consider the set of optimal solutions of the lexicographic optimization problem, X^1 is an efficient solution, one of the optimal solutions in this set.
  - The second lexicographic optimal solution X^2 corresponding to $lexmax\big(Z^2(x), Z^1(x)\big)$ is found in the same manner.

The indices sets corresponding to the regular variables D_0 and D_1, are calculated as follows:

1. For $i \in J_0$, the objects that are dominated by at least UB objects “in terms of efficiency” are grouped at the end; the values of these objects correspond to $x_i = 0$ in all efficient solutions.
2. For $i \in J_1$, the objects that dominate at least $n - LB$ objects “in terms of efficiency” are grouped at the first; the values of these objects correspond to $x_i = 1$ in all efficient solutions.

Table 1
Efficient solutions

i	1	2	3	4	5
X^1	1	1	0	1	0
X^2	1	1	1	0	0
X^3	1	0	0	1	1

Mathematically we write,

$$D_0 = \{i \in J_0 / |P(E^i)| \geq UB\} \tag{7}$$

such that $P(E^i)$ is the preferred set (Definition 10) and UB is the UB on the cardinality of a feasible solution for BOKP (Definition 3).

$$D_1 = \{i \in J_1 / |D(E^i)| \geq n - LB\} \tag{8}$$

such that $D(E^i)$ is the dominated set (Definition 11) and LB is the LB on the cardinality of a feasible solution for BOKP (Definition 3).

Generally, the exact values of the thresholds for which we could detect the regular variables indices sets D_0 and D_1 cannot be a priori identified, but a good approximation of these values are UB and $n - LB$, respectively, it is the reason why sometimes we miss a negligible number of efficient solutions among an important number of efficient solutions. We tried to remedy such cases by adding to the threshold values a certain additive value u (detail is given in the last paragraph of Section 5.5). We give a didactic example where the reduction procedure can miss one efficient solution:

$$(BOKP) \begin{cases} \max(Z_1) = 11x_1 + 5x_2 + 7x_3 + 13x_4 + 3x_5 \\ \max(Z_2) = 9x_1 + 2x_2 + 16x_3 + 5x_4 + 4x_5 \\ \qquad 4x_1 + 2x_2 + 8x_3 + 7x_4 + 5x_5 \leq 16 \\ \qquad x_i \in \{0, 1\} \end{cases} \tag{9}$$

An exact method provides the efficient solutions shown in Table 1.

Our procedure reduction detected the regular variables indices $D_1 = \{1\}$ and $D_0 = \{5\}$. The resolution of the reduced problem gives two efficient solutions X^1 and X^2, then we miss the third efficient solution X^3, because its fifth component is equal to 1.

4.1. Technical description

In this section, we give a technical description of our proposed algorithm.

Algorithm 1: Determine the regular variables indices by the object's efficiency

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Input:
$\downarrow C$: the profit matrix
$\downarrow w_i; i = 1, \cdots, n$: the weights
$\downarrow w$: the capacity
Output:
$\uparrow D = D_0 \cup D_1$: the set of regular variables indices
Initialization
$D_0 \leftarrow \varnothing, D_1 \leftarrow \varnothing$;
Find the efficiency vector of every objective
for $i \leftarrow 1$ ***to*** n **do**
  for $j \leftarrow 1$ ***to*** p **do**
    $e^j(i) \leftarrow \frac{C^{j(i)}}{w(i)}$;
Define the object's efficiency of the multi-objective problem ;
for $i \leftarrow 1$ ***to*** n **do**
  $E^i \leftarrow [e^1(i), e^2(i), ..., e^p(i)]$
Calculate LB and UB;
Calculate the supported efficient solutions $X^1, ..., X^p$;
($p \leftarrow 2$ for biobjective KP);
Define ;
$J_0 \leftarrow \{j \in \{1, \cdots, n\} \,|\, x_j^1 = x_j^2 = \cdots = x_j^p = 0\}$;
$J_1 \leftarrow \{j \in \{1, \cdots, n\} \,|\, x_j^1 = x_j^2 = \cdots = x_j^p = 1\}$;
$m_0 \leftarrow |J_0|, m_1 \leftarrow |J_1|$;
$D_0 \leftarrow \emptyset$;
for $i \leftarrow 1$ ***to*** m_0 **do**
  Calculate $P(E^{(J_0(i))})$;
  if $|P(E^{(J_0(i))})| \geq UB$ **then**
    $D_0 \longleftarrow D_0 \bigcup \{J_0(i)\}$
return D_0;
$D_1 \leftarrow \emptyset$;
for $i \leftarrow 1$ ***to*** m_1 **do**
  Calculate $D(E^{(J_1(i))})$;
  if $|D(E^{(J_1(i))})| \geq n - LB$ **then**
    $D_1 \longleftarrow D_1 \bigcup \{J_1(i)\}$
return D_1;

4.2. Didactic example

This example shows the advantage of the algorithm 1:

$$(BOKP)\begin{cases} \max(Z_1) = 11x_1 + 4x_2 + 5x_3 + 1x_4 + 7x_5 + 13x_6 + 4x_7 + 3x_8 \\ \max(Z_2) = 9x_1 + 8x_2 + 2x_3 + 2x_4 + 16x_5 + 5x_6 + 9x_7 + 4x_8 \\ \qquad 4x_1 + 3x_2 + 2x_3 + 1x_4 + 8x_5 + 7x_6 + 6x_7 + 5x_8 \leq 20 \\ \qquad x_i \in \{0, 1\} \end{cases} \tag{10}$$

Table 2
Efficient solutions

i	1	2	3	4	5	6	7	8
X^1	1	0	1	1	0	1	1	0
X^2	1	1	1	1	1	0	0	0
X^3	1	0	0	1	1	1	0	0

An exact method provides the efficient solutions shown in Table 2. The first, the fourth, and the eighth columns are the regular variables indices corresponding to $C_1 = \{1, 4\}$ and $C_0 = \{8\}$.

Applying the method stated in Jorge et al. (2008): We obtain:

- The data vectors :

$$v^1 = \begin{pmatrix} 11 \\ 9 \\ -4 \end{pmatrix} \quad v^2 = \begin{pmatrix} 4 \\ 8 \\ -3 \end{pmatrix} \quad v^3 = \begin{pmatrix} 5 \\ 2 \\ -2 \end{pmatrix}$$

$$v^4 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \quad v^5 = \begin{pmatrix} 7 \\ 16 \\ -8 \end{pmatrix} \quad v^6 = \begin{pmatrix} 13 \\ 5 \\ -7 \end{pmatrix} \quad v^7 = \begin{pmatrix} 4 \\ 9 \\ -6 \end{pmatrix} \quad v^8 = \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}.$$

- The $LB = 2$ and the $UB = 5$, respectively.
- $P(v^8) = \{1, 2\}$ and $D(v^1) = \{7, 8\}$ and $D(v^4) = \emptyset$ and according the properties in Jorge et al. (2008), we have: $|P(v^8)| \ngeq UB$ and $w_1 + w_2 + w_8 = 13 \not> w$ therefore nothing can be said, even if $x_8 = 0$ in all efficient solutions, also $8 - |D(v^1)| = 6 \nleq LB$ and $w_1 + w_2 + w_3 + w_4 + w_5 + w_6 \nleq w$, which does not determine the variable x_1, even if $x_1 = 1$ in all the efficient solutions.
  The same for the value x_4, we cannot determine it by the properties developed (Jorge et al., 2008).

As conclusion, the regular variables indices sets detected in Jorge et al. (2008) is given by $C_1^{'} = \emptyset$ and $C_0^{'} = \emptyset$.

Now, we apply the object's efficiency vectors technique:

$$E^1 = \begin{pmatrix} 2{,}75 \\ 2{,}25 \end{pmatrix} \quad E^2 = \begin{pmatrix} 1{,}33 \\ 2{,}66 \end{pmatrix} \quad E^3 = \begin{pmatrix} 2{,}5 \\ 1 \end{pmatrix}$$

$$E^4 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad E^5 = \begin{pmatrix} 0{,}87 \\ 2 \end{pmatrix} \quad E^6 = \begin{pmatrix} 1{,}85 \\ 0.71 \end{pmatrix} \quad E^7 = \begin{pmatrix} 0{,}66 \\ 1{,}5 \end{pmatrix} \quad E^8 = \begin{pmatrix} 0{,}6 \\ 0{,}8 \end{pmatrix}.$$

- Two supported efficient solutions are considered:

$$X^1 = (1, 0, 1, 1, 0, 1, 1, 0) \quad \text{and} \quad X^2 = (1, 1, 1, 1, 1, 0, 0, 0).$$

- The set J_0 and J_1 are defined by:

$$J_0 = \{8\} \text{ et } J_1 = \{1, 3, 4\}.$$

The dominated set is: $D(E^1) = \{3, 4, 5, 6, 7, 8\}$, as $|D(E^1)| \geq n - LB$ then $x_1 = 1$. The dominated set is $D(E^3) = \{6, 8\}$, we have $|D(E^3)| \ngeq n - LB$ therefore nothing can be said about x_3. The dominated set is $D(E^4) = \{5, 7, 8\}$, as $|D(E^4)| \ngeq n - LB$ nothing can be concluded about x_4. The dominant set is $P(E^8) = \{1, 2, 3, 4, 5, 7\}$, we have $|P(E^8)| \geq UB$ thus $x_8 = 0$.

Consequently, the procedure detected some regular variables indices $D_1 = \{1\}$ and $D_0 = \{8\}$ that could not been detected by Jorge et al. (2008).

5. Numerical experiments

5.1. The detection of sets regular variables

The instances from http://xgandibleux.free.fr/MOCOlib/instances/MOKP/ are used to validate our proposed algorithm.

The above algorithm, is implemented in the MATLAB environment. All the tests have been performed by a PC Intel® Pentium® M Processor 2.13 GHz. We use the CPLEX 12.2 library for solving the KP.

We adopt the following notations :

- $2KPn - r$: denotes a BOKP with n items and a ratio of the capacity to the total weights $r \in 10^{-2} \times [11, 91]$.
- $\mid C \mid$ is the number of regular variable indices obtained by an exact method given in the above site.
- $\mid C_J \mid$ is the number of regular variable indices given in Jorge et al. (2008).
- $\mid D \mid = \mid D_0 \bigcup D_1 \mid$ is the number of regular variable indices produced by our algorithm.
- T is the CPU time of our algorithm.

5.2. Results from instances 1A

This folder contains five data files that correspond to five biobjective $\{0, 1\}$-unidimensional KPs. The values (profit vectors and item weights) are uniformly generated. The instances count between 50 and 500 items. The ratio r lies in the range $[0.11, 0.92]$.

From Table 3, the resolution of the instance $2KP500 - 41$ gives 70 regular variables.

Table 3
Number of variables fixed to 0 and 1

Instances	$\mid C \mid$	$\mid C_J \mid$	$\mid D_0 \mid$	$\mid D_1 \mid$	$\mid D \mid$	$T(s)$
$2KP50 - 11$	31	10	28	0	28	0.59
$2KP50 - 50$	20	2	6	2	8	0.71
$2KP50 - 92$	48	28	1	34	35	0.5
$2KP100 - 50$	56	10	8	21	29	2.49
$2KP500 - 41$	Unknown	Unknown	60	10	70	6.97

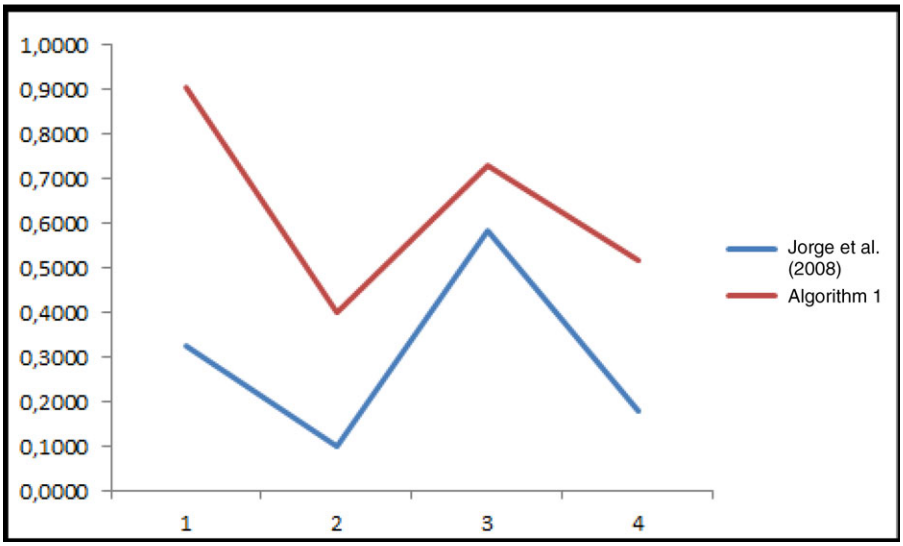

Fig. 1. The percentage of reduced variables.

The difference between both algorithms in percentage lies in the range 14.58% and 58.06% (see Fig. 1).

5.3. *Results from instances 1B*

Thirty data files corresponding to nine biobjective {0, 1}-unidimensional KPs are explored. The ratio is fixed to 0.5. Three variants *A*, *B*, and *C*, of nine instances are studied, where the profits and the weights in both first variants are generated from [1, 100] uniform distribution.

1. Class $1B/A$, the weights and the profits are uniformly generated in [1, 100] (see Table 4). The difference in percentage is also significant. It lies in the range 18.52% and 35.56% (see Fig. 2).

Table 4
Number of variables fixed to 0 or 1

Instances	$\mid C \mid$	$\mid C_J \mid$	$\mid D_0 \mid$	$\mid D_1 \mid$	$\mid D \mid$	$T(s)$
2*KP*50	27	11	6	10	16	0.52
2*KP*100	53	5	5	13	18	0.45
2*KP*150	83	18	14	24	38	0.95
2*KP*200	116	17	15	34	49	1.45
2*KP*250	139	18	15	39	54	2.23
2*KP*300	173	26	17	54	71	2.54
2*KP*350	213	31	21	58	79	3.75
2*KP*400	252	33	28	72	100	4.58
2*KP*450	270	27	35	88	123	5.47

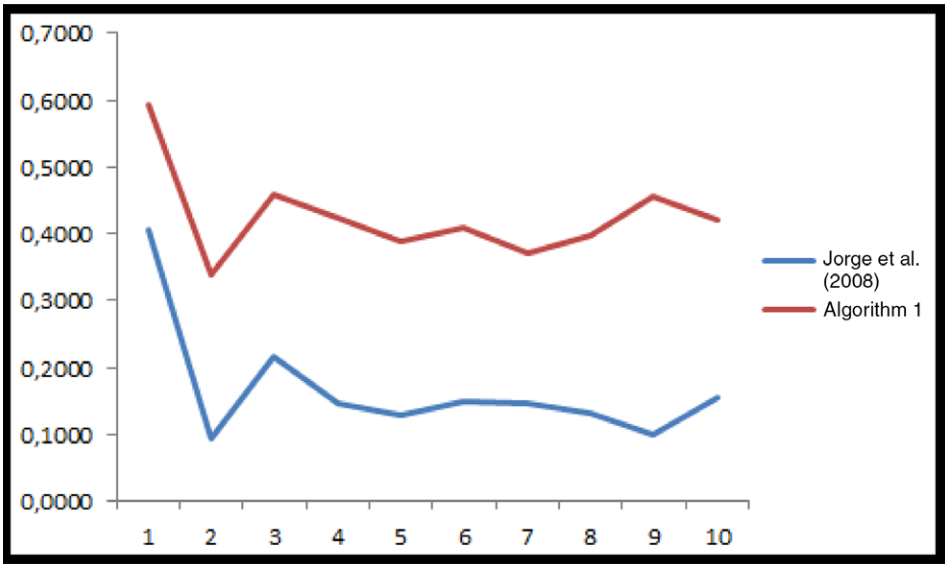

Fig. 2. The percentage of reduced variables.

Table 5
Number of variables fixed to 0 and 1

Instances	$\mid C \mid$	$\mid C_J \mid$	$\mid D_0 \mid$	$\mid D_1 \mid$	$\mid D \mid$	$T(s)$
$2KP50$	24	9	3	9	12	0.26
$2KP100$	51	4	7	14	21	0.52
$2KP150$	78	9	7	23	30	0.87
$2KP200$	108	15	11	25	36	1.26
$2KP250$	145	18	14	38	52	2.03
$2KP300$	180	32	18	51	69	2.23
$2KP350$	203	31	25	51	76	3.82
$2KP400$	249	39	29	64	93	4.87
$2KP450$	249	31	25	77	102	6.10

2. Class $1B/B$ created from 1B/A by replacing the second vector of profits by the first one in reverse order ($c_i^2 = c_{n-i+1}^1, \forall i$) (see Table 5). In this category, all the instances produce a clear difference (see Fig. 3).
3. Class $1B/C$, the profit vector components are uniformly generated in the intervals of lengths that do not exceed 10 % of the problem size ($[1, \lfloor \frac{n}{10} \rfloor]$) (see Table 6). This set of instances presents the same results, all instances produce a clear difference (see Fig. 4).

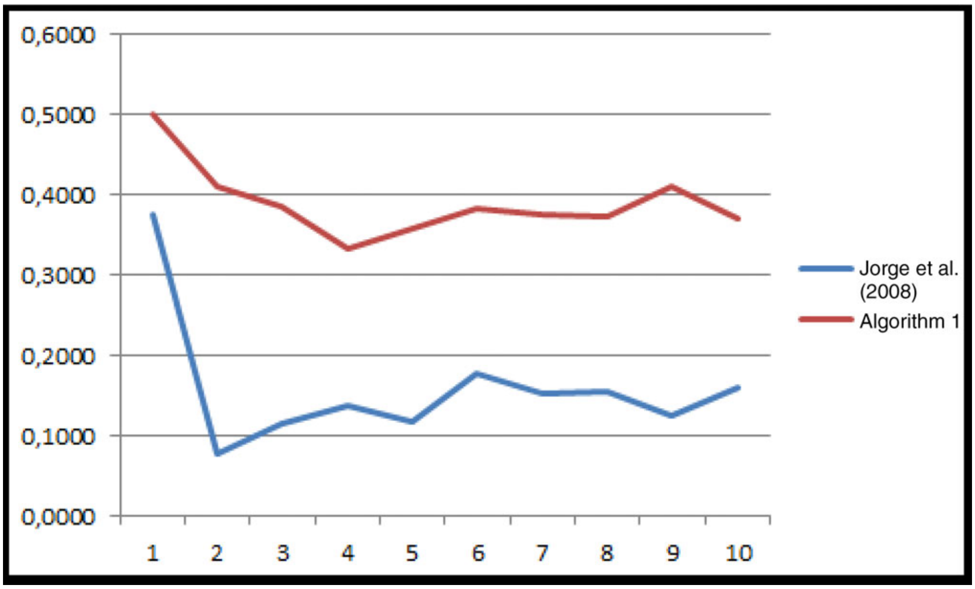

Fig. 3. The percentage of reduced variables.

Table 6
Number of variables fixed to 0 and 1

Instances	$\|C\|$	$\|C_J\|$	$\|D_0\|$	$\|D_1\|$	$\|D\|$	$T(s)$
$2KP50$	23	3	3	8	11	0.21
$2KP100$	61	11	11	17	28	0.63
$2KP150$	81	10	13	27	40	0.86
$2KP200$	105	19	10	28	38	1.45
$2KP250$	136	15	11	43	54	2.03
$2KP300$	186	37	28	46	74	2.86
$2KP350$	193	29	10	54	64	3.82
$2KP400$	239	47	41	71	112	4.77
$2KP450$	291	27	28	87	115	5.96

The above results lead us to the following conclusions:

1. A great similarity is observed between the class $1B/A$ and $1B/B$.
2. $D \subset C$, except the first instance for class $1B/C$; at one side, our procedure produces the index $31 \in D_1$ but $31 \notin C_1$. On the other side, all efficient solutions show that there exist only one efficient solution among 79 solutions, its component x_{31} is equal to zero while all other efficient solutions their component x_{31} is equal to 1; therefore, when solving the problem in our reduced feasible solution set, one can miss one solution among 79 efficient solutions.
3. $|C_J| < |D| < |C|$, the difference between $|D|$ and $|C_J|$ is significant.

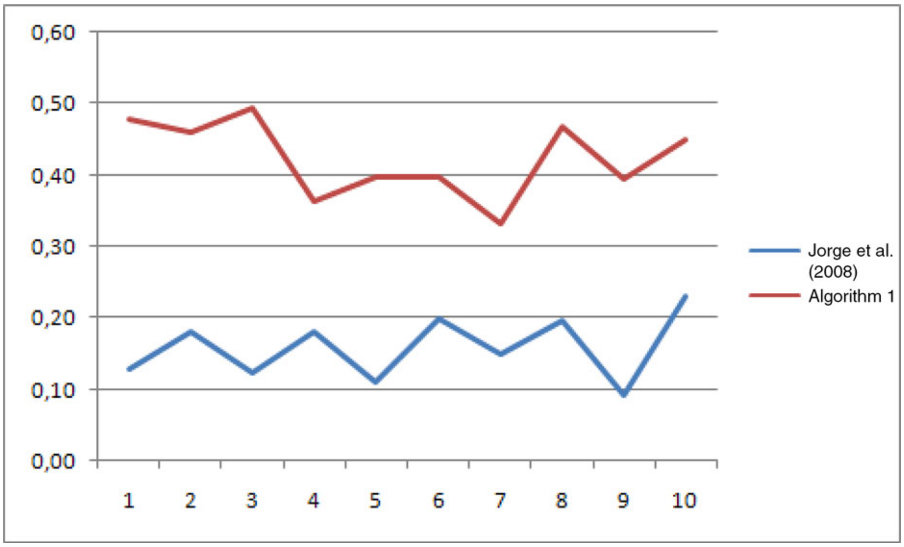

Fig. 4. The percentage of reduced variables.

4. The mean running time is acceptable, between 0.21 and 6.10 seconds.
5. For all instances, the size of problem is limited at 450.

5.4. *Results from instances 1C*

There are 38 data files corresponding to three series of biobjective {0, 1}-unidimensional KPs; uncorrelated denoted by UNCOR, weakly correlated denoted by WEAK, and strongly correlated named STRONG. The ratio is also fixed to 0.5.

1. UNCOR: the file is composed of 20 instances of 50 variables. The profits and the weights are uniformly generated; 10 of them lie in the range [1, 300] and the others are in the range [1, 1000] (see Table 7). Concerning this set of instances, the same results are found (see Fig. 5).
2. WEAK: Nine correlated instances of size between 50 and 450. The weights are uniformly generated in [1,1000]. The second vector of profits takes its values in range [111,1000]. The first vector of profits is randomly chosen in $[c2 - 100, c2 + 100]$ (see Table 8). Concerning the weak correlated instances, the proposed algorithm gives more then 50% regular variables, which means more than half of the dimension of the problem is reduced (see Fig. 6).
3. STRONG : Nine strongly correlated instances of size between 50 and 450 are used. The weights are uniformly generated in [1, 1000]. The second vector of profits takes its values in the range [1, 1000]. The first vector of profits is set equal to $w_j + 100$ (see Table 9). From the sixth entry ($2KP350 -> 2KP450$), for strongly correlated instance, we note that our approach produces regular variables but the other method (Jorge et al., 2008) does not (see Fig. 7).

Table 7
Number of variables fixed to 0 and 1

Instances	$\mid C \mid$	$\mid C_J \mid$	$\mid D_0 \mid$	$\mid D_1 \mid$	$\mid D \mid$	$T(s)$
2KP50	19	1	3	6	9	0.61
2KP50	25	5	7	6	13	0.51
2KP50	29	6	5	9	14	0.43
2KP50	28	4	5	6	11	0.57
2KP50	23	2	2	9	11	0.60
2KP50	26	4	5	7	12	0.43
2KP50	25	3	1	8	9	0.61
2KP50	27	5	6	8	14	0.76
2KP50	31	8	4	10	14	0.76
2KP50	27	6	5	8	13	0.55
2KP50	24	4	3	8	11	0.55
2KP50	30	9	5	9	14	0.49
2KP50	30	9	3	8	11	0.57
2KP50	25	5	6	5	11	0.55
2KP50	22	7	4	10	14	0.52
2KP50	25	3	6	6	12	0.42
2KP50	21	5	2	8	10	0.59
2KP50	33	12	8	10	18	0.60
2KP50	28	2	3	5	8	0.53
2KP50	22	4	2	10	12	0.77

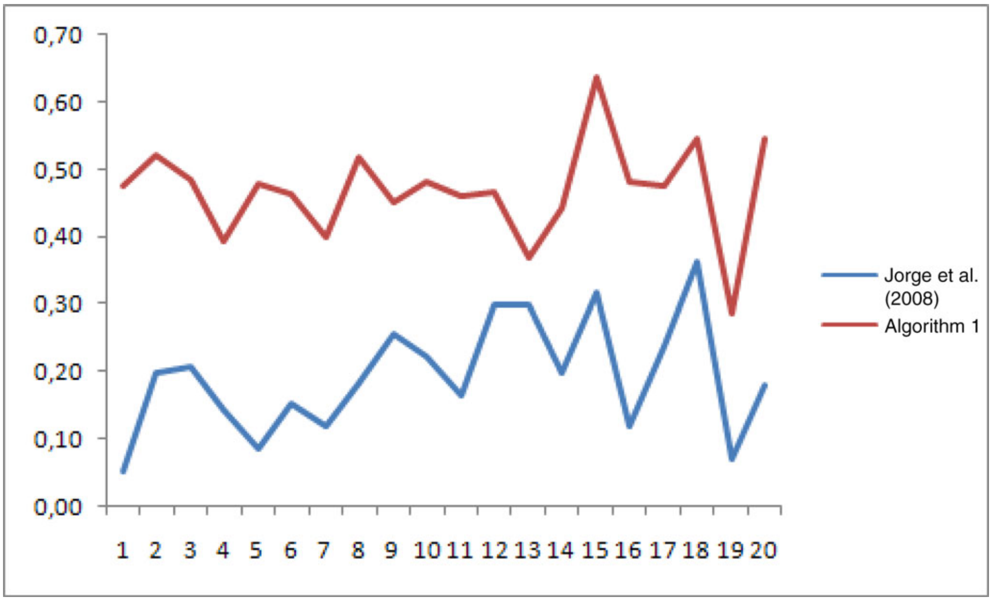

Fig. 5. The percentage of reduced variables.

Table 8
Number of variables fixed to 0 and 1

Instances	$\|C\|$	$\|C_J\|$	$\|D_0\|$	$\|D_1\|$	$\|D\|$	$T(s)$
$2KP50$	46	26	14	12	26	0.58
$2KP100$	100	40	28	27	55	1.17
$2KP150$	139	51	40	39	79	1.42
$2KP200$	197	64	55	53	108	1.97
$2KP250$	237	77	65	66	131	2.77
$2KP300$	281	94	79	79	158	3.88
$2KP350$	328	116	92	92	184	4.42
$2KP400$	373	133	104	107	211	6.69
$2KP450$	420	146	118	117	235	6.50

Finally, the results show that:

1. $D \subset C$, except the UNCOR class for the 11th and 13th instances; there exists only one index 39, 19 $\in D_1$, and D_0 but 39, 19 $\notin C_1$, and C_0, respectively.
   For example, the index 39 $\in D_1$ but 39 $\notin C_1$: the component $x_{39} = 1$ appears in the majority of efficient solutions ($x_{39} = 0$ appear in one solution among all efficient solutions).
2. $|C_J| < |D| < |C|$.
3. For the instances from type UNCOR and the instances WEAK, the mean running time is acceptable; between 0.42 and 0.77 seconds.
4. For the instances from type STRONG, our approach detects the regular variables set D for all instances, whereas the method in Jorge et al. (2008) does not detect any regular variable. The mean of execution time is acceptable 0.37 until 5.92.

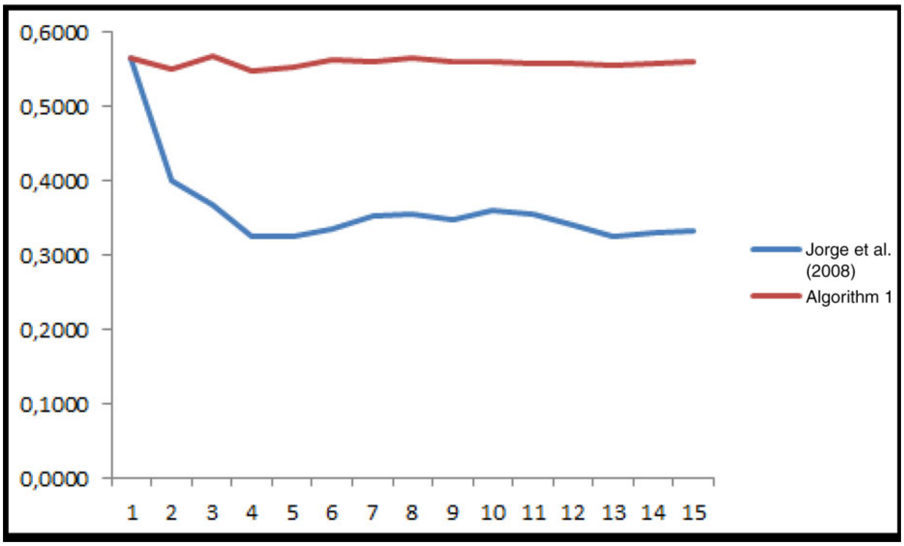

Fig. 6. The percentage of reduced variables.

Table 9
Number of variables fixed to 0 and 1

Instances	$\mid C \mid$	$\mid C_J \mid$	$\mid D_0 \mid$	$\mid D_1 \mid$	$\mid D \mid$	$T(s)$
$2KP50$	29	–	4	10	14	0.37
$2KP100$	69	–	8	21	29	0.75
$2KP150$	100	–	12	33	45	1.20
$2KP200$	140	–	16	44	60	1.87
$2KP250$	180	–	19	53	19	2.09
$2KP300$	212	–	23	63	86	3.27
$2KP350$	–	–	31	76	107	3.53
$2KP400$	–	–	34	85	119	4.87
$2KP450$	–	–	40	92	132	5.92

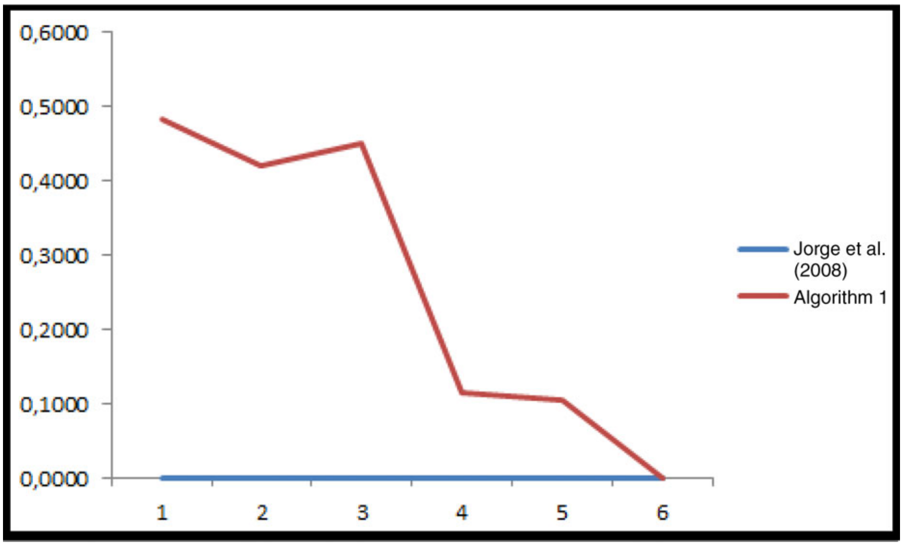

Fig. 7. The percentage of reduced variables.

5.5. *The impact of the regular variables for the running times of the problem resolution*

We used an exact method (Jahanshahloo et al., 2005) that optimizes the sum of both objective functions :

$Z^1(x) + Z^2(x)$; for each obtained optimal solution x_q^*, the admissible domain is reduced by adding three constraints:

$$Z^k(x) > Z^k\left(x_q^*\right) - Mt_{kq}, \quad k = 1, 2,$$

$$t_{1q} + t_{2q} \leq 1$$

and two (0, 1) variables t_{1q} and t_{2q}, the method provides at least one efficient solution. We note that solving the BOKP using this method (see Jahanshahloo et al., 2005) needs more computational effort, especially if the problem has an important number of efficient solutions. For this reason, the size of some instances is limited. The method is described as follows (see Jahanshahloo et al., 2005).

Consider the following BOKP:

$$(BOKP)\begin{cases} \max(Z^k) = \sum_{i=1}^{n} c_i^k x_i, & k = 1, 2 \\ \sum_{i=1}^{n} w_i x_i \leq \omega & \\ x_i \in \{0, 1\}, & i = 1, \ldots, n \end{cases} \tag{11}$$

For solving the problem (BOKP), according to Jahanshahloo et al. (2005), we first consider the following two single-objective KPs $KPk(k = 1, 2)$:

$$(KPk)\begin{cases} \max Z^k(x) \\ \sum_{i=1}^{n} w_i x_i \leq \omega \\ x_i \in \{0, 1\}, \quad i = 1, \ldots, n \end{cases} \tag{12}$$

Let $G_0 = \{x^*_{i_1}, x^*_{i_2}, \ldots, x^*_{i_\alpha}\}$ be the set of the optimal solution of problem (KPk) and $L_0 = \{i_1, i_2, \ldots, i_\alpha\}$.

- If the set G_0 is empty, then we solve the problem defined by

$$(SKP)\begin{cases} \max \sum_{k=1}^{2} Z^k(x) \\ \sum_{i=1}^{n} w_i x_i \leq \omega \\ x_i \in \{0, 1\}, \quad i = 1, \ldots, n \end{cases} \tag{13}$$

  Suppose that $G_0 = \{x^*_{i_1}, x^*_{i_2}, \ldots, x^*_{i_\beta}\}$ the set of the optimal solution of problem (SKP) stated in Equation (13) and $L_0 = \{i_1, i_2, \ldots, i_\beta\}$. Note that $x^*_q \in G_0$ with $q \in L_0$ is an efficient solutions of the problem (BOKP) defined in Equation (11).

- If the set G_0 is not empty, we determine all optimal solutions of the mono-objective defined by

$$(SKP1)\begin{cases}\max \sum_{k=1}^{2} Z^k(x) \\ Z^k(x) > Z^k\left(x_q^*\right) - Mt_{kq}, \quad k = 1, 2, q \in L_0 \\ \sum_{i=1}^{n} w_i x_i \leq \omega \\ t_{1q} + t_{2q} \leq 1; \qquad t_{1q}, t_{2q} \in \{0, 1\} \\ x_i \in \{0, 1\}, \qquad i = 1, \ldots, n \end{cases} \tag{14}$$

In case $t_{kq} = 1 (k = 1, 2)$, the constraint $Z^k(x) > Z^k(x_q^*) - Mt_{kq}$ is redundant, otherwise, it is not. The constraint $t_{1q} + t_{2q} \leq 1 (q \in L_0)$ implies that at least one of the constraints $Z^k(x) > Z^k(x_q^*) - Mt_{kq}, \quad k = 1, 2$ is not redundant (we can choose $\max_{1\leq k\leq 2}\{\sum_{i=1}^{n} c_i^k\}$ as an LB for M). Now, we suppose that $A = \{x^*_{i_{j+1}}, x^*_{i_{j+2}}, \ldots, x^*_{i_{j+l}}\}$ is the set of optimal solutions of the problem $(SKP1)$ stated in Equation (14), where $j = \alpha$ or β.

- If A is empty, then G_0 is the efficient set of the problem (BOKP).
- Else, the set of optimal solutions is $G_1 = G_0 \cup A$.

The other efficient solutions of the problem (BOKP), are obtained as follows : for each $x_q^* \in A$, we add three constraints to the problem $(SKP1)$:

$$Z^k(x) > Z^k\left(x_q^*\right) - Mt_{kq}, \quad k = 1, 2, \quad t_{1q} + t_{2q} \leq 1.$$

Moreover, we add two $(0, 1)$ variables to the problem $(SKP1)$, t_{1q} and t_{2q}. Consequently, the problem $(SKP1)$ can be written as:

$$(SKP2)\begin{cases}\max \sum_{k=1}^{2} Z^k(x) \\ Z^k(x) > Z^k\left(x_q^*\right) - Mt_{kq}, \quad k = 1, 2, \\ \sum_{i=1}^{n} w_i x_i \leq \omega \\ t_{1q} + t_{2q} \leq 1 \qquad q = i_1, i_2, \ldots, i_{j+1}, i_{j+2}, \ldots, i_{j+l} (j = \alpha \text{ or } \beta). \\ x_i \in \{0, 1\}, \qquad i = 1, \ldots, n \end{cases} \tag{15}$$

This process continues until the problem $(SKP2)$ stated in Equation (15) becomes infeasible. Finally, we conclude by some theorems used in the method (Jahanshahloo et al., 2005):

Theorem 1. *If $G_l = \{x^*_{1_l}, x^*_{2_l}, \ldots, x^*_{f_l}\}$ is the set of optimal solutions of lth problem obtained from the problem (KPk) equation, then at least one of these optimal solutions is an efficient solution for problem $(BOKP)$.*

Theorem 2. *An optimal solution of the problem (SKP) is an efficient solution for problem $(BOKP)$.*

The CPU time T_E of the exact method (Jahanshahloo et al., 2005) is compared to the CPU time T_R of our reduction strategy implemented within the same exact method. We adopt the following notations:

- $2KPn - r$: denotes a BOKP with n items and a ratio of the capacity to the total weights $r \in 10^{-2} \times [11, 91]$.
- $\mid D \mid = \mid D_0 \bigcup D_1 \mid$ is the number of regular variable indices produced by our reduction strategy.
- N_R denotes the size of the reduced problem calculated as follows $N_R = n - \mid D \mid$.
- T_E and M_E are, respectively, the CPU time and the number of efficient solutions of developed exact method (Jahanshahloo et al., 2005).
- T_R and M_R are, respectively, the CPU time and the number of efficient solutions of our reduction strategy adapted in the exact method (Jahanshahloo et al., 2005).
- $T = \frac{T_R}{T_E} \times 100\%$ is the percentage of the running times of reduction strategy adapted in the exact method T_R and the running times of the same exact method T_E.

1. **Results from instances type 1A**
   We note that, for the instances 1A, the running time concerning our reduction method is better than the running time of the exact method Jahanshahloo et al. (2005). When the problem has at most two efficient solutions, the running time is the same for both methods (e.g., $2KP50 - 92$).
   When the size of the problem is slightly large, (e.g., $2KP100 - 50$), full enumeration is used in the iterative exact method even if the reduction size was made ($n = 100 -> N_R = 71$) . For the latter reason, the line of these instances is empty ($*$) (see Table 10).
2. **Results from instances type 1C**
   We note that for all instances of UNCOR type, the running time for our reduction method is faster than the exact method (Jahanshahloo et al., 2005). The percentage of the running times T is significant between 65.30% until 97.66% (see Table 11).
3. **Results from instances type WEAK**
   We note that for instances of WEAK type, the running time for our reduction method is faster than the exact method (Jahanshahloo et al., 2005) for the majority of instances except the

Table 10
The difference time between the exact (Jahanshahloo et al., 2005) and reduction methods

	Exact method [6]		Our reduction method				
Instances	M_E	T_E	$\mid D \mid$	N_R	M_R	T_R	$T\%$
$2KP50 - 11$	43	85.47	28	22	42	51.30	60.02
$2KP50 - 50$	51	408.70	8	42	51	399.35	97.71
$2KP50 - 92$	2	0.48	35	15	2	0.48	100
$2KP100 - 50$	*	*	29	71	*	*	*

Table 11
The difference time between exact and reduction methods for UNCOR

Instances	Exact method [6]		Our reduction method				
	M_E	$T_E(s)$	$\mid D \mid$	N_R	M_R	$T_R(s)$	$T\%$
$2KP50$	59	425.40	9	41	59	399.03	93.80
$2KP50$	37	106.85	13	37	37	93.25	80.64
$2KP50$	39	115.63	14	36	39	112.93	97.66
$2KP50$	34	62.74	11	39	34	54.39	86.69
$2KP50$	40	186.25	11	39	40	127.86	68.64
$2KP50$	40	107.75	12	38	40	88.84	82.45
$2KP50$	38	119.76	9	41	38	103.14	86.12
$2KP50$	30	92.50	14	36	30	78.15	84.48
$2KP50$	33	50.10	14	36	33	37.09	74.03
$2KP50$	44	140.99	13	37	44	133.16	94.44
$2KP50$	56	353.51	11	39	56	336.58	95.21
$2KP50$	49	193.05	14	36	49	126.07	65.30
$2KP50$	27	21.93	11	39	26	18.44	84.08
$2KP50$	47	182.67	11	39	47	136.94	74.96
$2KP50$	65	766.56	14	36	65	678.10	88.46
$2KP50$	45	153.90	12	38	45	122.49	79.59
$2KP50$	62	679.80	10	40	62	460.92	67.80
$2KP50$	19	13.94	18	32	19	8.59	61.62
$2KP50$	34	63.52	8	42	34	44.66	70.30
$2KP50$	80	1736.94	12	38	80	1337.3	76.99

Table 12
The difference time between exact and reduction methods for WEAK correlated

Instances	Exact method [6]		Our reduction method				
	M_E	$T_E(s)$	$\mid D \mid$	N_R	M_R	$T_R(s)$	$T\%$
$2KP50$	3	8.79	26	24	2	0.58	6.59
$2KP100$	1	1.17	55	45	1	1.17	100
$2KP150$	8	5.43	79	71	8	4.00	73.66
$2KP200$	2	1.97	108	92	2	1.97	100
$2KP250$	7	10.98	122	118	7	9.97	90.80
$2KP300$	16	81.53	158	142	16	59.07	72.45
$2KP350$	25	193.87	184	166	25	106.65	55.01
$2KP400$	25	344.19	211	189	24	276.46	80.32
$2KP450$	–	–	235	215	35	873.71	–

instances $2KP100$ and $2KP200$, which the problem has, respectively, one and two efficient solutions. For the last instances $2KP450$, our reduction method gives the set of efficient solutions, but the exact method cannot because the academic version (cplex12.6) is limited (see Table 12).

In conclusion, Tables 2–12, show the impact of the reduction strategy time on global running time for solving the BOKP. The running time of the reduction strategy adapted in the exact method (Jahanshahloo et al., 2005) is compared with that of the same exact method without taking into account the reduction strategy, the results show the significant difference of CPU-time.

We note that, for all the instances, the sign " $-$ " signifies that the method does not provides a results, and the sign " $*$ " signifies that the method provides a full enumeration and therefore no results.

To analyze the performance of the method, we use the instances in the above section corresponding to the classes $1A$, $1B$, $1C$.

- The reduction algorithm is particularly efficient for all instances of type $1A$,$1B/A$, $1B/B$, WEAK, and STRONG.
- It should be noted that for some instances where the data depend on the size of the problem, for example, some instances of type $1B/C$ and UNCOR, the set of regular variables indices D_0 and D_1 are included in C_0 and C_1 (respectively), but there exist exactly one index included in D_0(resp D_1) not in C_0(resp C_1), which can cause a negligible miss of efficient solutions compared to an important set of efficient solutions.

For this type of instances, we proceed as follows:

Based on dominance Pareto relationships on the object efficiency; we sort increasingly the objects according to $|P(E^i)|$ (or we sort decreasingly the objects according to $|D(E^i)|$) as follows:

1. For $i \in J_0$, the objects that are dominated by a great number of objects (more than UB objects) (in terms of object's efficiency) are grouped at the end; where the ratios E^i, $i = 1, \ldots, n$ are very small (objects of lower efficiency) and the values of these objects corresponding to $x_i = 0$ in all efficient solutions.
2. For $i \in J_1$, the objects that dominate a great number of objects (more than $n - LB$ objects) (in terms of object's efficiency) are grouped at the beginning, where the ratios E^i, $i = 1, \ldots, n$ are important (objects of higher efficiency) and the values of these objects corresponding to $x_i = 1$ in all efficient solutions.

To ensure the great number, a certain value u is added in two latter cases as follows.

The sets D_0 and D_1 are calculated through:

$$D_0 = \left\{ i \in J_0 / |P(E^i)| \geq UB + u \right\} \tag{16}$$

$$D_1 = \left\{ i \in J_1 / |D(E^i)| \geq n - LB + u \right\} \tag{17}$$

where, u is an additional positive integer number.

Proposition 2. $u \in [0, \min(LB - 1, n - UB - 1)]$.

Proof. E^i can be dominated by E^j (in the sense of Pareto) a maximum $n - 1$ object and minimum 0 object such that

$0 \leq |P(E^i)| \leq n - 1$, and from (16) we have $|P(E^i)| \geq UB + u$.

Thus, $UB + u \leq |P(E^i)| \leq n - 1 \Longrightarrow UB + u \leq n - 1$ then

$$u \leq n - UB - 1. \tag{18}$$

E^i dominates E^j (in the sense of Pareto) maximum $n - 1$ object and minimum 0 object such that

$0 \leq |D(E^i)| \leq n - 1$ and from (17) we have $|D(E^i)| \geq n - LB + u$.

Thus, $n - LB + u \leq |D(E^i)| \leq n - 1 \Longrightarrow n - LB + u \leq n - 1$ then

$$u \leq LB - 1 \tag{19}$$

By (18) and (19), we have

$$u \leq \min(LB - 1, n - UB - 1)$$

then

$$u \in [0, \min(LB - 1, n - UB - 1)]$$

6. Conclusions

In this paper, a new algorithm reducing the size of biobjective {0, 1}-KP is presented. This reduction is based on data information concerning the object's efficiency to fix some variables to 0 or 1 of the problem before its resolution, this pretreatment is based on the research for two extreme supported efficient solutions and a relation of the object's efficiency dominance in the sense of Pareto.

The analysis of the previous section shows that the research space of regular variables and the problem's size is considerably reduced in a reasonable calculation time and the results obtained are very interesting compared to the results in Jorge et al. (2008). This pretreatment however, is acceptable in calculation time for all instances, and in addition, the dimension of the research space is halved or reduced more (e.g., the instances "STRONG").

We also note that for all the studied instances, the condition using LB and UB in Jorge et al. (2008) fixes almost none of the regular variables, but the developed method using LB and UB determines many regular variables and it is interesting to determine the exact value of additional number u for some instances of type "$1B/C$" and "UNCOR."

For problems with large size, we propose incorporating this pretreatment before or in parallel with a metaheuristics and search only in the reduced area. Adding such algorithm can give more results with cooperative procedures as well.

Extending this study to MKP and taking into account more than two objectives are our future research work.

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